

# Vortex Lattice Depinning vs. Vortex Lattice Melting: a pinning-based explanation of the equilibrium magnetization jump

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## Abstract

In this communication we argue that the Vortex Lattice Melting scenario fails to explain several key experimental results published in the literature. From a careful analysis of these results we conclude that the Flux Line Lattice (FLL) does not melt along a material- and sample-dependent boundary  $H_j(T)$  but the opposite, it de-couples from the superconducting matrix becoming more ordered. When the FLL depinning is sharp, the difference between the equilibrium magnetization  $M_{eq}(T, H)$  of the pinned and unpinned FLL leads to the observed step-like change  $\Delta M_{eq}(T, H)$ . We demonstrate that the experimentally obtained  $\Delta M_{eq}(T, H)$  can be well accounted for by a variation of the pinning efficiency of vortices along the  $H_j(T)$  boundary.

*Key words:* A. superconductors, D. flux pinning, D. phase transitions

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The phenomenological description of superconductors is based on the knowledge of their magnetic field-temperature ( $H - T$ ) phase diagram. The equilibrium behavior of conventional type-II superconductors in an applied magnetic field is well known: At fields below the lower critical field  $H_{c1}(T)$ , the superconductor is in the Meissner-Ochsenfeld phase in which surface currents screen the magnetic field from the interior of the sample. Above  $H_{c1}(T)$  the field penetrates the superconductor in the form of a lattice of vortices, the so-called Abrikosov vortex lattice. This flux-line-lattice (FLL) persists up to the upper critical field  $H_{c2}(T)$  where superconductivity vanishes in the bulk of the sample.

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In high-temperature superconductors (HTS), however, due to strong thermal fluctuations, a first-order phase transition of the FLL to a liquid-like state, the “melting” of the FLL, has been predicted to occur [1,2] well below  $H_{c2}(T)$ . Since then, an enormous amount of experimental and theoretical work has been done trying to find this, or other more sophisticated transitions and to extend the original theoretical treatment[3,4]. At the beginning of these research activities it was claimed that the melting transition of the FLL in HTS manifests itself at the damping peak of vibrating HTS in a magnetic field[5]. However, this interpretation was controversial [6]. We know nowadays that the damping peaks in vibrating superconductors attributed to the melting transition [5] can be explained quantitatively assuming thermally activated depinning and the diffusive motion of the FLL under a small perturbation generated by the vibration of the sample [7]. To find the true experimental evidence for the melting transition of the FLL is by no means simple: Because the FLL interacts with the superconducting matrix through pinning centers (atomic lattice defects, surface barriers, etc.) every property of the FLL one measures will be influenced by the pinning and, therefore, no direct and straightforward proof of the melting phase transition can be achieved.

Several years after the above cited first experimental attempt, a striking jump in the equilibrium magnetization in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  (Bi2212) single crystals has been measured using a SQUID[8] as well as sensitive micro-Hall sensors[9]. This magnetization jump, which was interpreted as a first-order transition of the FLL, lies at more than one order of magnitude lower fields than the thermally activated depinning line measured with vibrating superconductors[5,7]. These interesting results[8,9] are important because if the melting transition would be of the first order, a discontinuous change in the equilibrium magnetization  $M_{eq}(T, H)$  at the transition is expected. The step-like increase of  $M_{eq}(T, H)$  (a decrease in absolute value) with increasing magnetic field and temperature has been also observed in  $\text{YBa}_2\text{Cu}_3\text{O}_7$  (Y123) single crystals[10].

Apart from the clearly defined magnetization jump, another important fact was revealed by the experiments. It has been found that the line  $H_j(T)$  along which  $M_{eq}(T, H)$  jumps, and the temperature dependence of the field  $H_{\text{SMP}}(T)$  where an anomalous maximum in the width of the magnetization hysteresis loop takes place (the so-called “second magnetization peak” (SMP)), define a unique boundary on the  $H - T$  plane for a given sample,[11,12] demonstrating their intimate relationship. In order to account for this behavior, a second-order phase transition associated with the increase of the critical current density  $j_c(T, H)$  was suggested as the origin of SMP based on a pinning-induced disordering of the FLL[13–15].

Recently, a similar SMP occurring at a temperature dependent field  $H_{\text{SMP}}(T) \ll H_{c2}(T)$  was also measured in conventional Nb superconducting films[16,17]. The studies of the SMP performed on Nb films,[16,17] Bi2212 single crys-

tals,[18] as well as on a non-cuprate isotropic single crystalline  $\text{Ba}_{0.63}\text{K}_{0.37}\text{BiO}_3$  thick film[19] provide a clear evidence that the SMP is not related to a critical current enhancement, but originates from a thermomagnetic instability effect and/or a non-uniform current distribution,[20] leading to the “hollow” in  $M(H)$  at  $H < H_{\text{SMP}}(T)$ . In agreement with the models, the SMP vanishes in all these superconductors when the lateral sample size becomes less than  $\sim 100 \mu\text{m}$  [16–19]. Because  $100 \mu\text{m}$  is much larger than all relevant vortex-pinning-related characteristic lengths, the strong influence of the sample geometry on the SMP cannot be explained by a change of the pinning efficiency of vortices. Besides, the results[17,18] suggest that the interaction between vortices starts to dominate that between vortices and the matrix at  $H > H_{\text{SMP}}(T)$ . We note that the results obtained in Bi2212 crystals[18] are actually in good agreement with the second-order diffraction small-angle neutron scattering (SANS) experiments which revealed a well-ordered FLL at  $H > H_{\text{SMP}}(T)$  [21]. Moreover, the formation of a more ordered FLL with increasing temperature due to thermal depinning has been found in the above mentioned SANS experiments[21] at  $H > H_{\text{SMP}}(T)$  and for intermediate temperatures. A similar result was obtained by means of Lorentz microscopy in Bi2212 thin films near the low field - high temperature portion of the “irreversibility line”[22]. The high resolution SANS measurements, recently reported for Y123 crystals[23] also revealed a well defined FLL up to a field of 4 T and at low temperatures, i.e. above the  $H_{\text{SMP}}(T)$ -line measured in similar crystals[12].

Based on this experimental evidence and the intimate relationship between SMP- and the magnetization-jump-lines,[11,12] we propose that the jump in  $M_{eq}(T, H)$  results from a magnetic-field- and temperature-driven FLL depinning transition to a more *ordered* state of the FLL, effectively de-coupled from the atomic lattice.

There exist already experimental[24–26] as well as theoretical[27] works that show that the interaction of vortices with pinning centers *increases*  $|M_{eq}(T, H)|$ , indicating clearly that pinning influences the thermodynamic, equilibrium properties of superconductors. If the FLL depinning is sharp,[28,29] then one expects a step-like change of the equilibrium magnetization  $|\Delta M_{eq}(T, H)| = |M_{eq}^{dis}(T, H) - M_{eq}(T, H)|$  along the  $H_j(T)$  boundary. Here  $|M_{eq}^{dis}(T, H)| > |M_{eq}(T, H)|$  is the absolute equilibrium magnetization in the presence of the quenched disorder which is measured below  $H_j(T)$ . We note further that the sharpness of the vortex depinning onset, irrespectively of the underlying mechanism, manifests itself as a sudden increase in the electrical resistivity at  $H_j(T)$ , below which the vortex behavior is irreversible[30].

In what follows we present a phenomenological approach to describe the magnetization jump observed at the depinning transition. The equilibrium magnetization of an ordered, unpinned FLL in the London regime and neglecting

fluctuations[31] is given by the equation

$$M_{eq} = -\frac{\phi_0}{2(4\pi\lambda)^2} \ln(\eta H_{c2}/H), \quad (1)$$

where  $\lambda(T) \equiv \lambda_{ab}(T)$  is the in-plane London penetration depth,  $\phi_0$  is the flux quantum, and  $\eta$  is a parameter analogous to the Abrikosov ratio  $\beta_A = \langle |\Psi|^4 \rangle / \langle |\Psi|^2 \rangle^2$  ( $\Psi$  being the superconducting order parameter) that depends on the FLL structure[32]. We assume now that the  $\Delta M_{eq}$  results from a change of the parameter  $\eta$  due to a change in the vortex arrangement triggered by the interaction of the FLL with pinning centers. Hence, the magnetization jump can be written as

$$\Delta M_{eq} = \frac{\phi_0}{2(4\pi\lambda)^2} \ln(\eta^{dis}/\eta_0), \quad (2)$$

where the parameter  $\eta^{dis} \geq \eta_0$  is related to the strength of the quenched disorder, and  $\eta_0$  applies for the FLL above the  $H_j(T)$ . Therefore, it is reasonable to assume that  $\eta^{dis}$  is proportional to the critical current density  $j_c(T, H)$  which measures the vortex pinning strength. In fact, the correlation between  $\Delta M_{eq}$  and the pinning of the FLL has been observed experimentally[33]. We emphasise that although the relationship between  $\eta$  and  $\beta_A$  is unknown, one expects an increase of the parameter  $\eta$  in the vortex liquid state compared to that of the vortex solid, similar to the results obtained for  $\beta_A$  [34].

We show in Fig. 1 the jumps of the induction  $\Delta B(T, H)$  and of the magnetization  $4\pi\Delta M(T, H)$  measured along the  $H_j(T)$  boundaries in Bi2212 (a)[9] and Y123 (b)[10] single crystals, respectively. The difference in the behavior of  $\Delta M_{eq}(T, H)$  in Bi2212 and Y123 crystals (see Fig. 1) can be easily understood noting that  $H_j(T) < H^* = \phi_0/\lambda^2$  for Bi2212, whereas  $H_j(T) \gg H^*$  in the case of Y123. At fields  $H < H^*$  the FLL shear modulus exponentially decreases with field as  $c_{66} \simeq (\epsilon_0/\lambda^2)(H\lambda^2/\phi_0)^{1/4} \exp(-\phi_0/H\lambda^2)$ , whereas at  $H > H^*$  it is proportional to the field  $c_{66} \simeq (\epsilon_0/4\phi_0)H$  [29].

The exponential decrease of  $c_{66}$  with decreasing  $H_j$  (increasing  $T_j$ ) in the case of Bi2212, implies an enhancement of the interaction between vortices and the quenched disorder (pinning centers) which leads to an increase of  $\Delta M_{eq}$  increasing  $T_j$  (or decreasing  $H_j$ ), see Fig. 1(a). As temperature tends to the critical one  $T_c$ , the pinning of vortices vanishes. Therefore, above a certain temperature,  $\Delta M_{eq}(T, H)$  decreases with temperature and tends to zero.

In the case of Y123, however, and due to a relatively weak field dependence of  $c_{66}$  along the  $H_j(T)$  line, the reduction of the vortex pinning efficiency with temperature is the dominant effect which explains the monotonous decrease of  $\Delta M_{eq}(T, H)$  with temperature, see Fig. 1(b). We stress that the vanishing

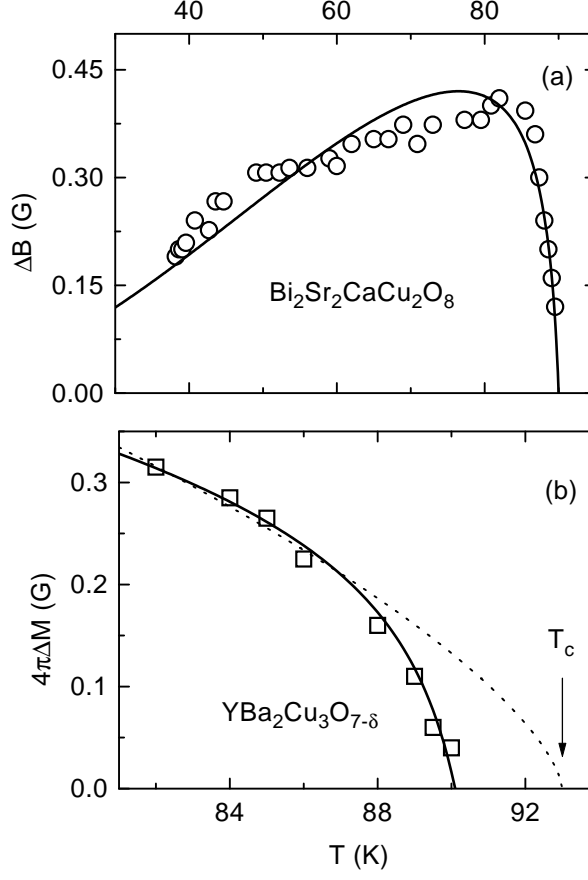


Fig. 1. The jump  $\Delta B$  in the induction (a) and equilibrium magnetization  $4\pi\Delta M$  (b) as a function of temperature measured for  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  (Bi2212) [9] and  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (Y123)[10] single crystals, respectively. The solid lines are obtained from Eq. (2) with  $\eta^{dis} = \eta_0[1 + aj_{c0}(1 - T/T_0)^n/(H^\alpha + H_0)]$ , with the fitting parameters  $n = 1, \alpha = 1.8, H_0 = 200 \text{ Oe}^{1.8}, T_0 = 90 \text{ K}, aj_{c0} = 9.3 \times 10^4 \text{ Oe}^{1.8}$  for Bi2212 and  $n = 1, \alpha = 1, H_0 = 3.5 \times 10^3 \text{ Oe}, T_0 = 90.1 \text{ K}, aj_{c0} = 2.6 \times 10^5 \text{ Oe}$  for Y123. Dotted line (b) corresponds to the equation  $4\pi\Delta M = 1.32(1 - T/T_c)^{2/3}(T_c = 92.9 \text{ K})$ , according to the theoretical result from melting theory [36].

of the magnetization jump ( $\Delta M_{eq}(T, H)$ ) at  $T_0 \simeq 90 \text{ K}$ , i.e. approximately 3 K below the superconducting transition temperature  $T_c = 92.9 \text{ K}$ [10] (similar result was obtained in another untwinned Y123 crystal[35]), can be also explained naturally by the effect of thermal fluctuations which smear out the pinning potential. On the other hand, the theory[36] based on the FLL melting hypothesis predicts  $\Delta M_{eq} \sim \phi_0/\lambda^2(T)$  (dotted line in Fig. 1(b)) which implies the vanishing of  $\Delta M_{eq}$  at  $T_c$ . While the FLL-melting theory[36] requires different approaches in order to explain  $\Delta M_{eq}(T, H)$  in Y123 and Bi2212, our analysis can equally well be applied to both weakly (Y123) and strongly (Bi2212) anisotropic superconductors.

In order to use Eq. (2) to calculate the magnetization jump we need the relationship between  $\eta^{dis}$  and the critical current density  $j_c(T, H)$  at the boundary  $H_j(T)$ , which is unknown at present. Nevertheless and as an illustration of our ideas we present here a simple fitting approach. The solid lines in Fig. 1(a,b) were obtained from Eq. (2) with  $\eta^{dis} = \eta_0[1 + aj_c(T, H)] = \eta_0[1 + aj_{c0}(1 - T/T_0)^n/(H^\alpha + H_0)]$  calculated at the boundary  $H_j(T)$ , where  $T_0$  corresponds to the temperature at which  $j_c = 0$ , and  $a, j_{c0}, H_0$  are model-dependent constants,  $n$  and  $\alpha$  are pinning related exponents. Note, that the here used  $j_c(T, H)$  is a rather general expression which reflects the well-known experimental fact that the critical current density generally decreases with temperature and increasing field. In our fits (solid lines in Fig. 1) we have set the exponents  $n = 1, \alpha = 1.8(1)$ , and used  $\lambda(T) = 250(1 - T/T_c)^{-1/3}$  nm ( $140(1 - T/T_c)^{-1/3}$  nm) for Bi2212 (Y123)[37]; other fitting parameters close to those used give also satisfactory fits.

Furthermore, within the here proposed physical picture we expect a reduction and ultimately the vanishing of  $\Delta M_{eq}(T, H)$  if by applying external driving forces one de-couples the FLL from the matrix. Such effect was observed in Bi2212 crystals, indeed[33]. Also, within our picture we expect that the FLL remains in a more ordered, metastable state if the sample is field cooled as compared to the zero-field-cooled state. Several published results have indicated such a behavior, both directly (see, e.g. Ref. [38]) and indirectly. Among them, the pioneer work[8] which demonstrates that the magnetization jump at the irreversibility line can be much larger in the zero-field-cooled sample compared to the field-cooled one, providing a clear evidence that  $\Delta M_{eq}(T, H)$  is essentially related to the competition between vortex-vortex and vortex-pinning centers interactions. Supporting the above ideas, the magnetic-field-driven transition from a disorder-dominated vortex state to a moving well-ordered FLL was observed in NbSe<sub>2</sub> low- $T_c$  layered superconductor [39].

Finally, we would like to point out that the equilibrium magnetization jump at a first-order depinning transition would imply the use of the Clausius-Clapeyron relation. However, we are not aware of any prediction on the entropy change at the depinning transition which we could use for comparison. Therefore, we believe that it has little sense to comment here on the use of the Clausius-Clapeyron equation at this stage of our study.

To summarise, based essentially on experimental facts we propose the magnetic-field and temperature-driven vortex-lattice-ordering transition as an alternative to the FLL melting scenario in high- $T_c$  superconductors. Simple arguments allowed us to account for the equilibrium magnetization jump associated with the FLL depinning transition. This transition awaits for a rigorous theoretical treatment.

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